# 서브밴드 분리에 근거한 새로운 근사 DCT 계산과 응용

정 성 환†


## 요 약

많은 영상 데이타 압축 응용분야에서 DCT는 높은 압축 성능으로 인하여 널리 잘 알려져 있다. 그러나 낮은 비트율에서는 인간의 시각에 거슬리는 블록 효과(block artifacts)가 생기는 문제점을 가지고 있다. 또한 실제적인 응용에 있어서, DCT 계수의 빠른 계산과 간단한 VLSI 구현도 중요한 과제이다. 따라서 블록 효과의 제거와 빠른 DCT 계산은 실제적인 연구의 대상이 된다.

본 논문에서는 수정된 DCT 계산 방법을 연구하였고, 이것은 빠른 계산과 함께 블록 효과를 효과적으로 제거할 수 있었다. 이러한 새로운 접근 방법을 낮은 비트율에서 영상의 부호화 및 복호화에 적용하였다. 실제 영상을 대상으로 실험한 결과, 제안된 방법을 통해 표준적인 방법에 비하여 성능을 개선할 수 있었다.


# A New Approximate DCT Computation Based on Subband Decomposition and Its Application

Sung-Hwan Jung†


## ABSTRACT

In many image compression applications, the discrete cosine transform(DCT) is well known for its highly efficient coding performance. However, it produces undesirable block artifacts in low-bit rate coding. In addition, in many practical applications, faster computation and easier VLSI implementation of DCT coefficients are also important issues. The removal of the block artifacts and faster DCT computation are therefore of practical interest.

In this paper, a modified DCT computation scheme was investigated, which provides a simple efficient solution to the reduction of the block artifacts while achieving faster computation. We have applied the new approach to the low-bit rate coding and decoding of images. Simulation results on real images have verified the improved performance of the proposed method over the standard method.


## 1. Introduction

In many signal information processing application, discrete transforms of finite-length sequences play an important role. Various discrete transform families have been advanced along with the fast algorithms for their computations. Of all such transforms, the discrete cosine transform(DCT) is generally recognized as the best and effective way to encode image information [1]. For compression of a highly correlated image, the DCT approaches the optimum performance of Karhunen Loeve transform(KLT) [2]. As a result, the DCT has been adopted in the JPEG coding

† 종신회원:창원대학교 전자계산학과
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standard for still images [3] and the MPEG coding standard for video images [4]. In general, DCT decorrelates the data being transformed so that the most of its energy is packed in a few of its transform coefficients. In image coding applications, an image is divided into a set of continuous small size sub-image blocks, typically of size 8×8, and the DCT of each block is then computed. To achieve high compression ratio, the transform coefficients with large magnitudes are quantized while those with small magnitudes, which are mostly in the high frequency band, are either coarsely quantized or discarded entirely with little distortion. However, in very low bit rate coding, the decoded image exhibits severe block artifacts which are often undesirable. A number of researchers have investigated the reduction of block artifacts [5-8]. In addition, there have been efforts to develop fast DCT computation algorithms [9-12]. However, in most of the work reported so far, the problem of block artifact reductions and the development of fast algorithms have been considered separately.

In fact, it has been demonstrated that image compression carried out on the sub-images obtained by a subband decomposition can be more effective than compressing the full band image [13]. Most of so called subband coding schemes have multirate filter banks to get subband decomposition. The design of these filter banks for perfect reconstructionis is also another research area[14]. Therefore, conventional subband DCT coding means the preprocessing of filter banks and then the standard DCT processing[15, 16].

In this paper, however, we develop a new DCT computation scheme based on the subband decomposition of DCT without the filter banks like the subband DCTs[17, 18]. As it will be shown, this approach do provide a simultaneous solution to both block artifact reductions and fast computation mentioned above.

We organized this paper as follows. First, in Section 2 our subbanded DCT(SB-DCT) computation scheme is introduced. Next, in Section 3 a partial-band analysis of the SB-DCT, that can be used for the efficient computation of the approximate values of the dominant DCT samples, is provided. In Section 4 we apply the fast approximate DCT computation scheme to image coding within the framework of JPEG. In Section 5 we demonstrate through simulation results that the SB-DCT based coding scheme shows less block artifacts while being much faster than standard DCT coding scheme. Section 6 contains conclusions.

## 2. Subband Decomposition of DCT

The DCT of an N point data sequence x(n), n = 0, 1, ... N − 1 is defined as in Eq(1)[1].

$$C(k)=\sum_{n=0}^{N-1} 2x(n)\cos\left(\frac{(2n+1)\pi k}{2N}\right),\quad 0\le k\le N-1 \quad (1)$$

Now, a length-N input sequence $x(n)$, with N being even, can be decomposed into two sub-sequences $x_l(n)$ and $x_h(n)$ of length N/2 each:

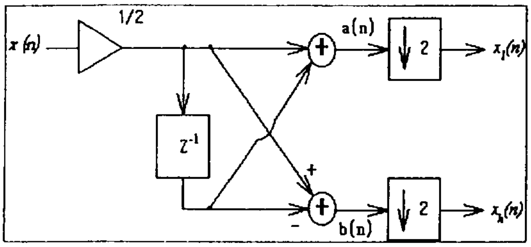
$$x_l(n)=\frac{1}{2}\{x(2n)+x(2n+1)\},$$
$$x_h(n)=\frac{1}{2}\{x(2n)-x(2n+1)\},\quad n=0,1,\cdots,\frac{N}{2}-1. \quad (2)$$

The original sequence $x(n)$ can be reconstructed from the subsequences $x_l(n)$ and $x_h(n)$ by means of the inverse relationship

$$x(2n)=x_l(n)+x_h(n),$$
$$x(2n+1)=x_l(n)-x_h(n),\quad n=0,1,\cdots,\frac{N}{2}-1. \quad (3)$$

A physical interpretation of the generation of the subsequences $x_l(n)$ and $x_h(n)$ is given in Figure 1 from which it is evident that the subsequence $x_l(n)$ is the down-sampled version of the lowpass filtered sequence a(n), and the subsequence $x_h(n)$ is the down-sampled version of the highpass filtered sequence b(n).

Substituting Eq. (3) into Eq. (1), we can rewrite Eq.




(Fig. 1) Subsequence by a 2-band decomposition.

(1) as in Eq(4)[17,18]

$$C(k)=\sum_{n=0}^{\frac{N}{2}-1} 2x(2n)\cos\left(\frac{(4n+1)\pi k}{2N}\right)+\sum_{n=0}^{\frac{N}{2}-1} 2x(2n+1)\cos\left(\frac{(4n+3)\pi k}{2N}\right)$$

$$=2\cos\left(\frac{\pi k}{2N}\right)\sum_{n=0}^{\frac{N}{2}-1} 2x_l(n)\cos\left(\frac{(2n+1)\pi k}{N}\right)+2\sin\left(\frac{\pi k}{2N}\right)\sum_{n=0}^{\frac{N}{2}-1} 2x_h(n)\sin\left(\frac{(2n+1)\pi k}{N}\right),$$

$$k=0,1,\cdots,N-1, \qquad (4)$$

or, equivalently as

$$C(k)=2\cos\left(\frac{\pi k}{2N}\right)\bar{C}_l(k)+2\sin\left(\frac{\pi k}{2N}\right)\bar{S}_h(k),\quad k=0,1,\cdots,N-1, \qquad (5)$$

where

$$\bar{C}_l(k)=\begin{cases} C_l(k), & 0\le k\le \frac{N}{2}-1,\\ 0, & k=\frac{N}{2},\\ -C_l(N-k), & \frac{N}{2}+1\le k\le N-1,\end{cases}$$

and

$$\bar{S}_h(k)=\begin{cases} S_h(k), & 0\le k\le \frac{N}{2}-1,\\ 1, & k=\frac{N}{2},\\ S_h(N-k), & \frac{N}{2}+1\le k\le N-1,\end{cases}$$

with $C_l(k)$ denoting the (N/2)-point DCT of $x_l(k)$, and $S_h(k)$ denoting the (N/2)-point DST(discrete sine transform) of $x_h(k)$. The computation of the N-point DCT using Eq. (5) requiring the computation of an (N/2)-point DCT and an (N/2)-point DST will be called the subbanded DCT.

The computational scheme outlined by Eq. (5) can be repeated now by replacing the computation of the two (N/2)-point transforms, $C_l(k)$ and $S_h(k)$, assuming N/2 is even, with expressions involving (N/4)-point transforms as follows:

$$C_l(k)=2\cos\left(\frac{\pi k}{N}\right)\sum_{n=0}^{\frac{N}{4}-1} 2x_{ll}(n)\cos\left(\frac{(2n+1)2\pi k}{N}\right)+2\sin\left(\frac{\pi k}{N}\right)\sum_{n=0}^{\frac{N}{4}-1} 2x_{lh}(n)\sin\left(\frac{(2n+1)2\pi k}{N}\right)$$

$$=2\cos\left(\frac{\pi k}{N}\right)\bar{C}_{ll}(k)+2\sin\left(\frac{\pi k}{N}\right)\bar{S}_{lh}(k),\quad k=0,1,\cdots,\frac{N}{2}-1, \qquad (6a)$$

$$S_h(k)=2\cos\left(\frac{\pi k}{N}\right)\sum_{n=0}^{\frac{N}{4}-1} 2x_{hl}(n)\sin\left(\frac{(2n+1)2\pi k}{N}\right)-2\sin\left(\frac{\pi k}{N}\right)\sum_{n=0}^{\frac{N}{4}-1} 2x_{hh}(n)\cos\left(\frac{(2n+1)2\pi k}{N}\right)$$

$$=2\cos\left(\frac{\pi k}{N}\right)\bar{S}_{hl}(k)-2\sin\left(\frac{\pi k}{N}\right)\bar{C}_{hh}(k),\quad k=0,1,\cdots,\frac{N}{2}-1, \qquad (6b)$$

where

$$\bar{C}_{ll}(k) = \begin{cases} C_{ll}(k), & 0 \le k \le \frac{N}{4} - 1, \\ 0, & k = \frac{N}{4}, \\ -C_{ll}(\frac{N}{2} - k), & \frac{N}{4} + 1 \le k \le \frac{N}{2} - 1, \end{cases}$$

$$\bar{C}_{hh}(k) = \begin{cases} C_{hh}(k), & 0 \le k \le \frac{N}{4} - 1, \\ 0, & k = \frac{N}{4}, \\ -C_{hh}(\frac{N}{2} - k), & \frac{N}{4} + 1 \le k \le \frac{N}{2} - 1, \end{cases}$$

$$\bar{S}_{lh}(k) = \begin{cases} S_{lh}(k), & 0 \le k \le \frac{N}{4} - 1, \\ 1, & k = \frac{N}{4}, \\ S_{lh}(\frac{N}{2} - k), & \frac{N}{4} + 1 \le k \le \frac{N}{2} - 1, \end{cases}$$

and

$$\bar{S}_{hl}(k) = \begin{cases} S_{hl}(k), & 0 \le k \le \frac{N}{4} - 1, \\ 1, & k = \frac{N}{4}, \\ S_{hl}(\frac{N}{2} - k), & \frac{N}{4} + 1 \le k \le \frac{N}{2} - 1, \end{cases}$$

with $C_{ll}(k)$ and $C_{hh}(k)$ denoting the (N/4)-point DCTs of $x_{ll}(x)$ and $x_{hh}(n)$ defined by:

$$x_{ll}(n) = \frac{1}{2}\{x_l(2n) + x_l(2n+1)\},$$

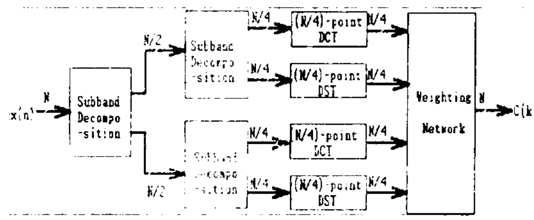
$$x_{hh}(n) = \frac{1}{2}\{x_h(2n) - x_h(2n+1)\}, \quad n = 0, 1, \cdots, \frac{N}{4} - 1. \tag{7}$$

and $S_{lh}(k)$ and $S_{hl}(k)$ denoting the (N/4)-point DSTs of $x_{lh}(n)$ and $x_{hl}(n)$ defined by:

$$x_{lh}(n) = \frac{1}{2}\{x_l(2n) - x_l(2n+1)\},$$

$$x_{hl}(n) = \frac{1}{2}\{x_h(2n) + x_h(2n+1)\}, \quad n = 0, 1, \cdots, \frac{N}{4} - 1. \tag{8}$$

Equation (6) can be repeated by replacing the computation of the (N/4)-point transforms with the expressions involving (N/8)-point transforms. The process can be continued until very short DCTs and DSTs are employed as indicated Figure 2.




(Fig. 2) A four-band SB-DCT algorithm.

## 3. Approximate Computation of DCT

The input information has energy concentrated in certain frequency bands in most practical applications. Assuming that the remaining frequency bands have negligible energy contributions, discarding of the calculations corresponding to the components in these bands makes the computation process simple. It results in a fast approximate DCT computation of the dominant DCT samples. As we shall demonstrate later, the approximate DCT samples represent the original information reasonably close with negligible distortion.

### 3.1 Half-Band Approximation of SB-DCT

Let us assume that most energy of the information is in the frequency range $k \in \left\{0, 1, \cdots, \frac{N}{2} - 1\right\}$. In this this case, we can carry out a half-band transformation with the approximated low-pass component and compute the approximate DCT coefficients according to:

$$\hat{C}(k) = \begin{cases} 2\cos\left(\frac{\pi k}{2N}\right)\bar{C}_l(k), & k \in \left\{0, 1, \cdots, \frac{N}{2} - 1\right\}, \\ 0, & otherwise. \end{cases} \tag{9}$$

In fact, in many image coding applications, most of relevant information in images is in the low frequencies. Therefore, the above approximation to the DCT coefficients obtained by neglecting $x_h(n)$, the high frequency components of $x(n)$ in Eq.(4), is reasonable. This approach is also intuitively satisfactory, as in the primary band of interest, the following inequalities hold:

$$\left|\cos\left(\frac{\pi k}{2N}\right)\right| > \left|\sin\left(\frac{\pi k}{2N}\right)\right|, \quad |\bar{C}_l(k)| \gg |\bar{S}_h(k)|. \tag{10}$$

Thus, a simple reasonable approximation of overall SB-DCT can be obtained by discarding the second term in Eq. (5) containing the contribution by the high frequency components.

### 3.2 Approximate IDCT computation

In a similar manner, like the forward DCT, we can compute the approximate inverse DCT (IDCT) of a given set of N DCT coefficients. The inverse discrete cosine transform (IDCT) of N point data sequence x (n), is defined as

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} \zeta(k) C(k) \cos\left(\frac{(2n+1)\pi k}{N}\right),$$

$$0 \le n \le N-1, \tag{11}$$

where

$$\zeta(o) = \frac{1}{2}, \quad \zeta(k) = 1 \qquad \text{for } 1 \le k \le N-1. \tag{12}$$

For example, if the dominant samples of the DCT are in the range of $0 \le k \le \frac{N}{2} - 1$, we can safely set $C(k)=0$, $k > \frac{N}{2} - 1$. As s result, Eq. (11) can be replaced with

$$\hat{x}(n) = \frac{1}{N}\sum_{k=0}^{\frac{N}{2}-1} \zeta(k) \hat{C}(k) \cos\left(\frac{(2n+1)\pi k}{N}\right),$$

$$0 \le n \le N-1, \tag{13}$$

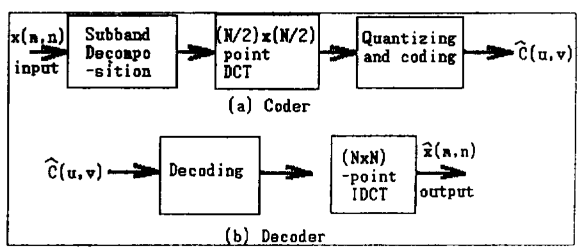
thus reducing the IDCT computational complexity to about half of that needed in the original one according to Eq. (11)[17, 18].

## 4. Image Coding Applications

In general, subband decompositions with $(N/2^{\mu}) \times (N/2^{\mu})$-point SB-DCT can be carried out with the number of stages $\mu$ depending on the applications.

### 4.1 Half-Band SB-DCT

The proposed coding scheme based on an approximate 2-D DCT computation using an half-band subband decomposition ($\mu=1$) is shown in Figure 3. First, after the first subband stage, the input sub-image block x(m, n) of size N×N is decomposed into 4 subimages using a 2-D 2×2 Hadamard and the sub-image corresponding to the half-band low-low (LL) frequency components is kept. Next, an (N/2)×(N/2)-point SB-DCT is computed from the low-low sub-image and the approximate SB-DCT coefficients $\hat{C}(u, v)$ are saved, quantized and Huffman coded under the JPEG framework. In the receiver, the coded SB-DCT coefficients are decoded and the restored image $\hat{x}(m, n)$ is recovered through an (N×N)-point IDCT.




(Fig. 3) Block diagram of the proposed coding system based on half band SB-DCT.

## 5. Simulation Results and Discussion

The proposed SB-DCT scheme has been applied to the coding of test image, LENA of size 512×512.

The 2-D SB-DCT has been implemented using the C language on a Sun/Sparc 1+ workstation. We have compared the performance of our proposed SB-DCT based coding system with that of the conventional DCT based JPEG coding for various average bit rates in the range 0.5−0.2 bpp. For the comparison of both schemes we used N=16 for the SB-DCT based method and N=8 for the standard DCT based method while employing the same JPEG quantization table in both cases. For the approximate DCT computation using the SB-DCT scheme we used a single stage decomposition ($\mu=1$).

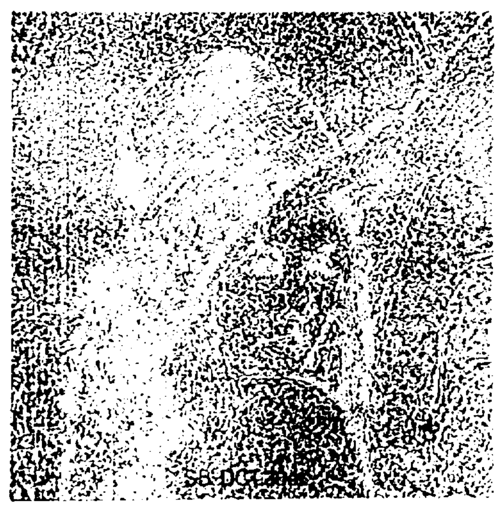
Table 1 shows the PSNR and the total time needed to develop the coded image for LENA for various values of the average bit rate. It can be seen from this table that both the SB-DCT and DCT based coding scheme exhibit similar performances with respect to the PSNR, but the SB-DCT based method is over 2 times faster than the conventional DCT based method. Because it computes only dominant coefficients and all the rest of coefficients are filled with zero on Eq. (9).

〈Table 1〉 Performance comparison of SB-DCT and DCT based coding of LENA image.

| | PSNR[dB] | | Time[sec] | |
|---|---|---|---|---|
| method<br>bpp | Proposed SB-DCT | Standard DCT | Proposed SB-DCT | Standard DCT |
| 0.50 | 34.11 | 34.49 | 38.4 | 85.5 |
| 0.45 | 33.98 | 34.15 | 38.4 | 85.5 |
| 0.40 | 33.75 | 33.74 | 38.3 | 85.3 |
| 0.35 | 33.53 | 33.27 | 38.3 | 85.2 |
| 0.30 | 33.20 | 32.61 | 38.1 | 85.1 |
| 0.25 | 32.86 | 31.71 | 38.1 | 85.0 |
| 0.20 | 32.35 | 30.28 | 38.1 | 85.0 |

Figures 4(a) and 4(b) show, respectively, the decoded LENA image obtained from the coded image at 0.3 bpp developed via the SB-DCT based approach and its associated error image. Figures 5(a) and (b) depict the corresponding images for the standard DCT based approach at the same bit rate. As can be seen, at low bit rates, the standard DCT based approach shows visible block artifacts, whereas, the SB-DCT based approach shows a more acceptable quality with almost no visible blocking effects.

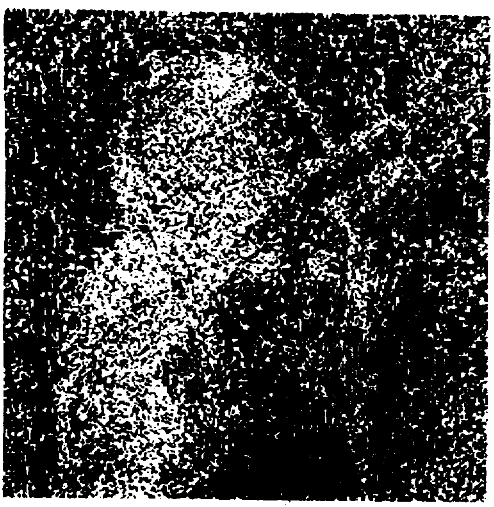
(a) SB-DCT



(b) error

(Fig. 4) The decoded version of the SB-DCT based JPEG coded(0.30bpp) image and its error image for the LENA image.

(a) DCT



(b) error

(Fig. 5) The decoded version of the DCT based JPEG coded(0.30bpp) image and its error image for the LENA image.

## 6. Conclusions

Based on a subband decomposition of the DCT definition, a new algorithm for the computation of DCT has been defined. This algorithm, unlike other well-known algorithms, permits approximate fast computation of dominant DCT samples without preprocessing such as multirate filter banks if the information energy is essentially confined to a small band. A detailed analysis of the new algorithm for the computation of all or partial DCT samples have been included.

The proposed method has been applied to the compression of real images to verify its performance in coding applications. The simulation results show that this approach yields acceptable quality with considerably less visible block artifacts over the standard JPEG method while being over two times faster. Various alterntives to the SB-DCT computation scheme are outlined which can be exploited to provide a trade off between image quality and computational complexity.

It should be pointed out here in the computer simulations carried out for comparing the performance of the SB-DCT with that of the conventional DCT, no attempt have been made to optimize the basic DCT program. As the same program is used for both computations, the relative computational efficiency of the SB-DCT with respect to the conventional DCT is expected to remain essentially the same, independently of the actual DCT program being used.

Further research efforts are concentrated in the anaysis and comparision with JPEG standard DCT using an optimized DCT code and applying this scheme to image sequences and finally to the real world.

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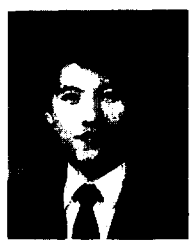

**정 성 환**

1975년 경북대학교 전자공학과(공학사)
1986년 전자계산기 기술사
1988년 경북대학교 대학원 영상정보처리(공학박사)
1992년 정보처리 기술사(전자계산 조직응용)
1992년~1994년 Univ. of California(UCSB) Post-Doc.
1983년~1985년 한국전자통신연구소(응용 S/W 연구실)
1988년~현재 창원대학교 전자계산학과 부교수
관심분야: 영상정보처리, 멀티미디어, 신경망