Alamouti 공간시간부호의 성능분석을 위한 closed-form BER 표현

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요 약

Alamouti 공간시간부호(Space Time Coding)은 UMTS-WCDMA의 표준의 한 부분으로 제안되고 연구되고 있다. 그러나 지금까지 알려진 바로는 이 유명한 부호의 closed-form 비트 에러율(BER)에 대한 공식은 연구된 적이 없다. 본 논문에서는 시뮬레이션을 통한 성능 검증이 시간 소비를 방지하기 위하여 coherent BPSK로 변조된 데이터를 이용하여 시스템 성능 분석시 참고할 수 있는 BER 성능을 수식으로 유도하였다.

키워드 : 공간시간부호화, AWGN, 레일리 페이딩

Closed-form BER expressions for performance of Alamouti STC

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ABSTRACT

Alamouti STC (Space-Time Code) is a part of the UMTS-WCDMA standard. However, up to the best of our knowledge no closed-form BER formula for this famous code exists. Evaluating its performance through simulations is time-consuming and therefore, there should be analytical BER graphs to serve as a reference which are derived in this paper for coherently BPSK-modulated data.

Key Words : STC, AWGN, Rayleigh Fading

1. Introduction

In wireless networks, signal fading arising from multi-path propagation is a particularly severe channel impairment that can be mitigated through the use of diversity [1] which is provided using temporal, frequency, polarization, and spatial resources. Among them, spatial diversity which relies on the principle that signals transmitted from geographically separated transmitters, and/or to geographically separated receivers, experience fading that is independent has been intensively researched recently. This kind of diversity technique usually uses a class of special codes called space-time codes [2] whose performance was proved to be very good in the flat Rayleigh fading channel by simulation programs. The first STC was suggested by Alamouti [3] with simple structure and high code rate (equal to 1), that is, there is no bandwidth sacrifice or equivalently, the application of STC remains the bandwidth as that of uncoded data. As a result, it is selected as a part of the UMTS-WCDMA standard [4]. However, its performance is only exposed through computer simulation which requires plenty of time. Therefore, the analytical BER expressions for this famous code should be established to serve as a reference. This is our motivation to use a mathematical approach so as to formulate probability of error formulas under the flat Rayleigh fading channel plus AWGN (Additive White Gaussian Noise). In final equations, we examine its BER for both cases of two propagation paths to the receive antenna : symmetric (same fading power) or asymmetric (different fading power).

The rest of this paper is organized as follows. The BER expressions are deduced in part 2. Part 3 presents the simulation and numerical results as well as the elaborative discussions of all possible channel conditions and finally, the paper is closed in part 4.

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2. Formula Construction

Alamouti STC for two transmit antennas is represented by a transmission matrix

$$STC = \begin{pmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{pmatrix}$$

where x_1 and x_2 are two consecutive MPSK-modulated symbols having the same amplitude at the input of STC encoder; $(\cdot)^*$ denotes complex conjugate operator.

The signal transmission on two transmit antennas is processed as follows. At the first time slot, x_1 and x_2 simultaneously are sent on antenna *1* and *2*, respectively. Then, $-x_2^*$ and x_1^* continue to be transmitted on antenna *1* and *2* at the second time slot.

2.1 Channel model

The flat fading channel is usually assumed for most spatial diversity systems in which path gains α_i from transmit antenna i to receive antenna are modeled as samples of independent zero-mean complex Gaussian random variables (r.v.'s) with variance σ_i^2, and are constant during two-symbol durations but change over longer intervals.

2.2 Receiver

Consider the case of one receive antenna. The received signal being a superposition of signals from two transmit antennas attenuated by fading and corrupted by noise is given by

$$\begin{aligned} r_1 &= x_1\alpha_1 + x_2\alpha_2 + n_1 \\ r_2 &= -x_2^*\alpha_1 + x_1^*\alpha_2 + n_2 \end{aligned} \qquad (1)$$

where r_1 and r_2 are the received signals in the 1^{st} and 2^{nd} time-slots; n_1 and n_2 are independent zero-mean complex Gaussian r.v.'s with variance σ^2.

Assuming coherent detection, maximum likelihood decoding can be achieved based only on linear processing at the receiver [3]. As a result, the symbols x_1 and x_2 are estimated by

$$x_1' = r_1\alpha_1^* + r_2^*\alpha_2 \qquad (2)$$

$$x_2' = r_1\alpha_2^* - r_2^*\alpha_1 \qquad (3)$$

Substituting r_1 and r_2 from Eq. (1) into Eqs. (2)-(3), we obtain

$$x_1' = \left(|\alpha_1|^2 + |\alpha_2|^2\right)x_1 + N_1 \qquad (4)$$

$$x_2' = \left(|\alpha_1|^2 + |\alpha_2|^2\right)x_2 + N_2 \qquad (5)$$

where

$$N_1 = n_1\alpha_1^* + n_2^*\alpha_2 \qquad (6)$$

$$N_2 = n_1\alpha_2^* - n_2^*\alpha_1 \qquad (7)$$

Eqs. (4)-(5) show that STC provides exactly performance as the 2-level receive maximum ratio combining.

For coherent BPSK modulation, the symbols x_1 and x_2 are detected by

$$\bar{x}_1 = sign\left(\mathrm{Re}\left(x_1'\right)\right) = sign\left(\lambda x_1 + \bar{n}_1\right) \qquad (8)$$

$$\bar{x}_2 = sign\left(\mathrm{Re}\left(x_2'\right)\right) = sign\left(\lambda x_2 + \bar{n}_2\right) \qquad (9)$$

where

- sign($\cdot$) : signum function
- Re($\cdot$) : real part of a complex number
- $\bar{n}_i = \mathrm{Re}(N_i)$, i=1, 2
- $\lambda = |\alpha_1|^2 + |\alpha_2|^2 = x + y$

Since α_i are zero-mean complex Gaussian r.v.'s with variance σ_i^2, x and y have exponential distribution with mean value of σ_i^2; that is,

$$f_x(x) = \lambda_x e^{-\lambda_x x} \qquad f_y(y) = \lambda_y e^{-\lambda_y y}$$

where

$$\lambda_x = 1/\sigma_1^2 \qquad \lambda_y = 1/\sigma_2^2 \qquad x = |\alpha_1|^2 \qquad y = |\alpha_2|^2$$

and x, $y \geq 0$; $f_x(x)$, $f_y(y)$ are pdfs of r.v. x and y, respectively.

The pdf of λ, hence, is expressed as

$$\begin{aligned} f_\lambda(\lambda) &= \int_{-\infty}^{\infty} f_x(x) f_y(\lambda - x)dx \\ &= \int_0^{\lambda} \lambda_x e^{-\lambda_x x}\lambda_y e^{-\lambda_y(\lambda - x)}dx \end{aligned}$$

$$= \begin{cases} \frac{\lambda_x \lambda_y}{\lambda_x - \lambda_y}\left[e^{-\lambda_y \lambda} - e^{-\lambda_x \lambda}\right] & ,\lambda_x \neq \lambda_y \\ a^2 e^{-a\lambda}\lambda & ,\lambda_x = \lambda_y = a \end{cases}$$

Moreover due to the assumption of the mutual independence of n_i, N_1 and N_2 are zero-mean complex Gaussian r.v.'s with the same variance ς^2, given the channel realizations :

$$\varsigma^2 = \left(|\alpha_1|^2 + |\alpha_2|^2\right)\sigma^2 = \lambda\sigma^2$$

As a consequence, $\bar{n}_i$ are also zero-mean Gaussian r.v.'s with the variance $\varsigma^2/2$.

From Eqs. (8)-(9), it is found that x_1 and x_2 are attenuated and corrupted by the same fading and noisy level and as a result, their probability of error is equal. Therefore, the probability of error can be calculated through BER of x_1. Moreover, since x_1 is BPSK-modulated and transmitted equally likely, the BER conditioned on the channel realizations is given by

$$P_e = P(\bar{n}_1 > \lambda|\lambda) = \int_\lambda^\infty \frac{1}{\sqrt{2\pi\varsigma^2/2}} \exp\left(-\frac{\bar{n}_1^2}{2\pi\varsigma^2/2}\right) d\bar{n}_1 = Q\left(\sqrt{\frac{2\lambda}{\sigma^2}}\right)$$

where

$$Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{y^2}{2}\right) dy = \frac{1}{\pi}\int_0^{\pi/2} \exp\left(\frac{-x^2}{2\sin^2\phi}\right) d\phi \qquad (10)$$

The second equality in Eq. (10) follows from the alternate Q-function representation in [6, Eq. (4.2)].

Finally, average BER is computed by averaging P_e over the parameter λ

$$P_{eAVG} = \int_0^\infty P_e f_\lambda(\lambda) d\lambda = \int_0^\infty Q\left(\sqrt{\frac{2\lambda}{\sigma^2}}\right) f_\lambda(\lambda) d\lambda$$

2.3 Asymmetric case

The asymmetric scenario happens when fading level of one of the propagation paths to the receive antenna is different from the other path. In this case, we have $\lambda_x \neq \lambda_y$ and therefore,

$$P_{eAVG} = \int_0^\infty Q\left(\sqrt{\frac{2\lambda}{\sigma^2}}\right) \frac{\lambda_x \lambda_y}{\lambda_x - \lambda_y}\left[e^{-\lambda_y \lambda} - e^{-\lambda_x \lambda}\right] d\lambda$$

By changing variable of the integration $\upsilon = \lambda/\sigma^2$, we obtain

$$P_{eAVG} = \frac{\lambda_x}{\lambda_x - \lambda_y}\int_0^\infty Q\left(\sqrt{2\upsilon}\right)\sigma^2 \lambda_y e^{-\sigma^2\lambda_y \upsilon} d\upsilon - \frac{\lambda_y}{\lambda_x - \lambda_y}\int_0^\infty Q\left(\sqrt{2\upsilon}\right)\sigma^2 \lambda_x e^{-\sigma^2\lambda_x \upsilon} d\upsilon$$

Each component integral in the above expression is calculated in the Appendix, thus

$$P_{eAVG} = \frac{\lambda_x}{2(\lambda_x - \lambda_y)}\left[1 - \sqrt{\frac{1}{1+\sigma^2\lambda_y}}\right] - \frac{\lambda_y}{2(\lambda_x - \lambda_y)}\left[1 - \sqrt{\frac{1}{1+\sigma^2\lambda_x}}\right]$$

$$= \frac{1}{2(\sigma_2^2 - \sigma_1^2)}\left(\sigma_2^2\left[1 - \sqrt{\frac{1}{1+\frac{\sigma^2}{\sigma_2^2}}}\right] - \sigma_1^2\left[1 - \sqrt{\frac{1}{1+\frac{\sigma^2}{\sigma_1^2}}}\right]\right) \qquad (11)$$

2.4 Symmetric case

This is the case that both paths of similar quality to the destination, that is, $\lambda_x = \lambda_y = a = 1/\sigma_1^2$. Hence, we obtain

$$P_{eAVG} = \int_0^\infty Q\left(\sqrt{\frac{2\lambda}{\sigma^2}}\right) a^2 \lambda e^{-a\lambda} d\lambda$$

By changing the variable of the integration $z = a\lambda$ and letting $\beta = 1/(a\sigma^2) = 1/(\sigma^2/\sigma_1^2)$, the probability of error is reduced to

$$P_{eAVG} = \int_0^\infty Q\left(\sqrt{2\beta z}\right) z e^{-z} dz$$

$$= \int_0^\infty \left[\frac{1}{\pi}\int_0^{\pi/2} \exp\left(\frac{-2\beta z}{2\sin^2\phi}\right) d\phi\right] z e^{-z} dz$$

$$= \frac{1}{\pi}\int_0^{\pi/2}\left[\int_0^\infty z \exp\left(-\left[\frac{\beta}{\sin^2\phi} + 1\right] z\right) dz\right] d\phi$$

$$= \frac{1}{\pi}\int_0^{\pi/2}\left(\frac{\beta}{\sin^2\phi} + 1\right)^{-2} d\phi$$

$$= \frac{1}{\pi}\int_0^{\pi/2}\left(\frac{\sin^2\phi}{\sin^2\phi + \beta}\right)^2 d\phi$$

Using [6, Eq. (5A.4b)], we rewrite the above as

$$P_{eAVG} = \frac{(1-\chi)^2(2+\chi)}{4} \quad (12)$$

where

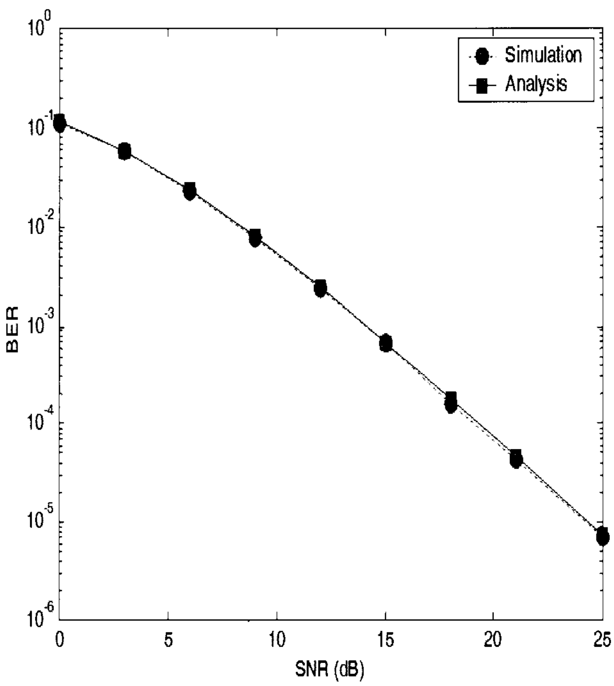
$$\chi = \sqrt{\frac{\beta}{1+\beta}} = \sqrt{\frac{1}{1+1/\beta}} = \frac{1}{\sqrt{1+\left(\sigma^2/\sigma_1^2\right)}}$$

Eqs. (11)-(12) are closed-form BER expressions for the STC with 2-transmit antennas and 1-receive antenna associated with coherent BPSK modulation.

3. Numerical results

In the following presented simulation and numerical results, the signal-to-noise ratio SNR is defined as $SNR = (\sigma_1^2 + \sigma_2^2)/\sigma^2 = (1+\delta)\sigma_1^2/\sigma^2$, where $\delta = \sigma_2^2/\sigma_1^2$. Moreover, the channel state information is assumed to be perfectly known.

3.1 Symmetric case

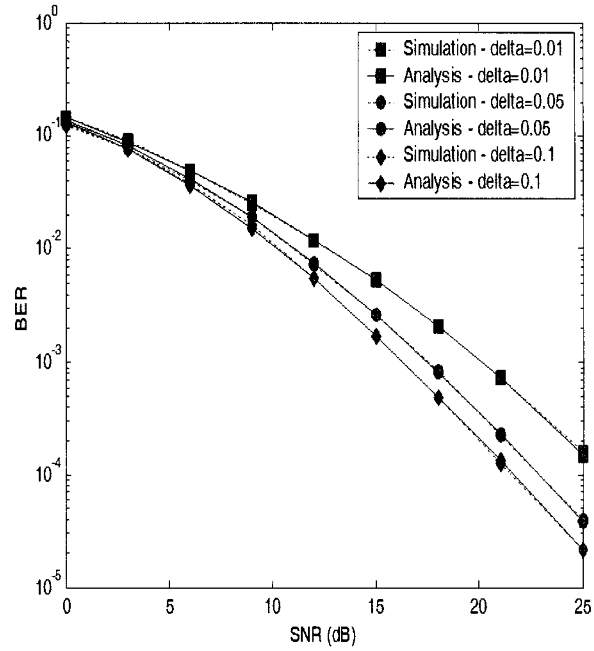


(Fig. 1) BER performance of Alamouti STC for the symmetric case

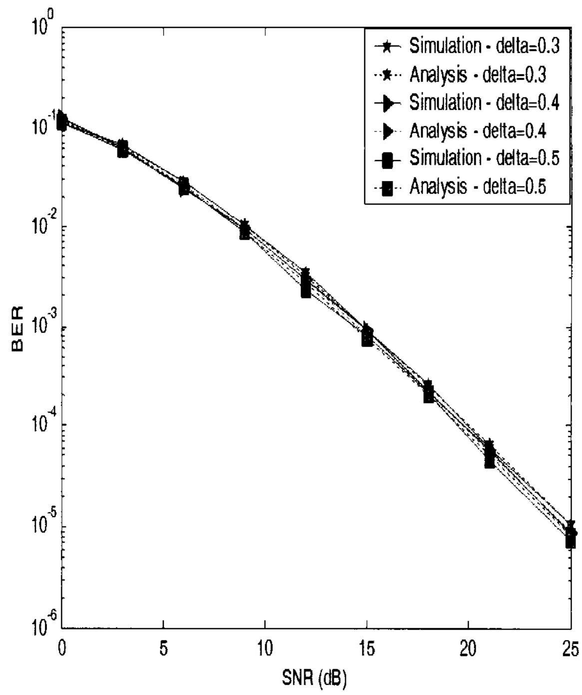
(Fig. 1) shows the BER performance of the Alamouti STC as the quality of both channels from transmit antennas to receive one is similar ($\sigma_1^2 = \sigma_2^2$). It is obvious that there is no difference between analysis (Eq. 12) and simulation. Therefore, the mathematical analysis is demonstrated to be completely exact.

3.2 Asymmetric case

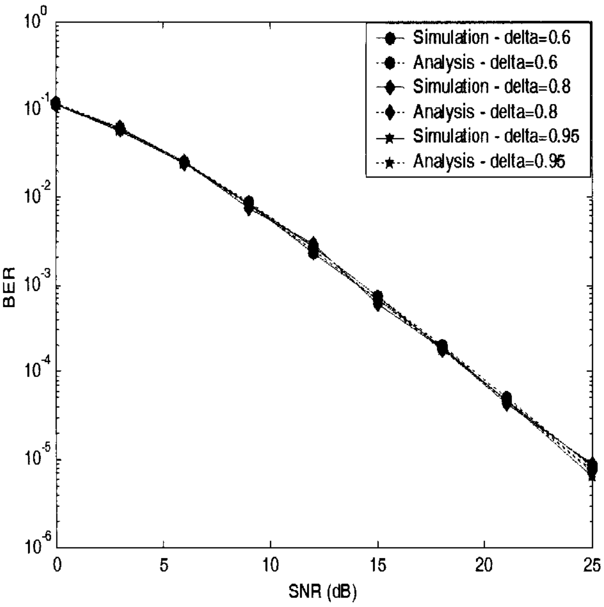
We consider the asymmetric scenario because in order for the path gains to be independent, the antenna elements should be spaced sufficiently apart [7]. Then the signals arriving at the receive antenna may experience the fading with different average power or equivalently, δ is unequal to *1*.



(a)



(b)



(c)

(Fig. 2) BER performance of Alamouti STC for the different δ's

(Fig. 2) investigates the influence of δ on the error probability of the Alamouti STC. It is realized that the smaller the value of δ, the worse the performance. Specifically, the case of $\delta=0.01$ suffers a SNR loss of approximately 3dB compared to that of $\delta=0.05$ at the same target BER of 10^{-4}. This is because if one of two propagation paths to the receiver is severely faded, the transmit diversity level is reduced and thus, the performance degradation happens. (Fig. 2) also reveals that STC achieves the best performance when the symmetry of channels to the destination is reached. Moreover, the analyzed BER curves Eq. (11) perfectly agree with the simulated ones. This again proves that the mathematical derivation is absolutely precise.

4. Conclusion

The exact BER formulas for analyzing the performance of one of the most important STCs, Alamouti STC, are derived. The simulation results verified the validity of those expressions. Therefore, they serve well as a reference for quickly estimating the BER performance without time-consuming simulations.

Appendix

In this appendix, we derive the following closed-form expression

$$P_{e\_1} = \int_0^{\infty} Q\left(\sqrt{2q}\right)\lambda_q e^{-q\lambda_q} dq$$

Using the alternate Q-function representation in [6, Eq. (4.2)], we obtain

$$\begin{aligned} P_{e\_1} &= \int_0^{\infty}\left[\frac{1}{\pi}\int_0^{\pi/2}\exp\left(\frac{-2q}{2\sin^2\phi}\right)d\phi\right]\lambda_q \exp(-q\lambda_q)dq \\ &= \frac{\lambda_q}{\pi}\int_0^{\pi/2}\left[\int_0^{\infty}\exp\left(-\left[\frac{1}{\sin^2\phi}+\lambda_q\right]q\right)dq\right]d\phi \\ &= \frac{\lambda_q}{\pi}\int_0^{\pi/2}\left[\frac{1}{\sin^2\phi}+\lambda_q\right]^{-1}d\phi \\ &= \frac{1}{\pi}\int_0^{\pi/2}\frac{\sin^2\phi}{\sin^2\phi+1/\lambda_q}d\phi \\ &= \frac{1}{2}\left(1-\sqrt{\frac{1}{1+\lambda_q}}\right) \end{aligned}$$

The last equality is achieved by using [6, Eq. (5A.9)].

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